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A Liquefaction Analysis Method Based on Finite Deformation Theory using a Cyclic Elasto-plastic Model for Sand

Fusao Oka*, Takeshi Kodaka, Ryo Morimoto, and Nobuyo Kita

Department of Civil Engineering, Kyoto University, Yoshida-Honmachi, Kyoto 606-8501, Japan
e-mail: foka@nakisuna.kuciv.kyoto-u.ac.jp

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Abstract

From the field investigation of liquefaction and ground deformation due to Hyogoken Nambu Earthquake, we have found that large deformation occurred in the liquefied ground. In order to simulate a liquefaction induced large deformation a liquefaction analysis method that can describe large deformation is necessary. The aim of the present paper is to propose a liquefaction analysis method by an elasto-plastic constitutive model based on finite deformation theory using both *u*-*p* (displacement - pore pressure) and *u*-*w*-*p* (displacement - relative displacement - pore pressure) formulation techniques.

1 Introduction

From the field observation of liquefaction and ground deformation due to Hyogoken Nambu Earthquake, we have found that that very larger deformation occurred in the liquefied ground, in particular near structures such as quay wall [1][2]. Observed data show that liquefaction analysis method that can describe large deformation is necessary. The aim of the present study is to propose a liquefaction analysis method based on finite deformation theory in order to well simulate a liquefaction induced deformation.

In order to numerically predict liquefaction phenomena of water saturated soil, transient analysis of two-phase material is particular importance. Several methods have been proposed by researchers (*e.g.* Zienkiewicz et al. 1980 [3], Zienkiewicz and Bettes 1982 [4]). Most of them are based on Biot's theory of fluid saturated porous media [5]. In the full formulation of dynamics of fluid saturated soil, unknown variables are the displacements of solid phase u, the displacements of fluid phase U, and the pore water pressure p. For compressible fluid, p can be eliminated by the constitutive law of fluid. Hence the formulation is considered as u-U formulation (Prevost 1982 [6], Zienkiewics and Shiomi 1984 [7]). In this formulation, U can be replaced by relative displacement between fluid and solid displacements w. This formulation is called u-w ($w=n(U-u)$) formulation (Ghaboussi and Wilson 1972 [8]). If the relative acceleration is small enough compared with the solid acceleration, w can be neglected and u-p formulation is sufficient for the analysis. Many numerical codes have been developed based on the u-p formulation (Zienkiewicz et al. 1980 [3], Zienkiewicz and Bettes 1982 [4], Simon et al. 1984 [9], Aubry and Moderessi 1989 [10], Oka et al. 1994 [11]). The other formulation is called u-w-p formulation. This formulation is classified as full formulation for compressible and incompressible pore fluid. The question is what formulation is preferable for the specific practical problems. An attempt for the answer to this question is given by Zienkiewicz et al. (1980) [3]. Zienkiewicz et al. (1980) reported that although it is necessary to adopt u-w-p formulation for the problem with higher frequency wave and higher permeability coefficient, u-p formulation is sufficient for the problems in some rage of frequency and permeability coefficient based on the one-dimensional analytical study of water saturated elastic material. Few studies are carried out for the performance of u-w-p formulation using an elastio-plastic model for soil skeleton. Lin and Borja (2000) [12] and Lin (2000) [13] have done simulation of liquefaction of elasto-plastic soil using u-w-p formulation. In the present paper we have carried out a numerical simulation of elasto-plastic soil using u-w-p formulation to study validity and limit of u-p formulation.

2 Finite Element Formulation

In the numerical analysis, a cyclic plasticity model for sand (Oka et al., 1999 [14]) and Biot's type two phase mixture theory are used. For the discretizaion of Biot' type governing equations for two-phase mixture in space, finite element method are adopted with the virtual work theorem.

For the dynamic analysis, there are several methods such as u-p (displacement - pore water pressure) formulation and u-w-p (displacement - relative acceleration - pore water pressure) formulation etc. (Zienkiewicz et al. 1980 [3]). In the u-p formulation method, acceleration of soil skeleton and pore water pressure are taken as independent variables, on the other hand, in the u-w-p formulation, acceleration of soil skeleton, relative acceleration (difference between accelerations of soil skeleton and pore fluid) and pore pressure are independent variables.

For the finite element formulation, we employed updated-Lagrangian method with Jaumann stress rate tensor.

We consider the equation of motion for water saturated soil and the continuity equation for pore fluid at time $t+\Delta t$. The effective stress concept is used in the analysis as:

$$T_{ij} = T'_{ij} + p\delta_{ij} \tag{1}$$

where T'_{ij} is the effective stress tensor, T_{ij} is the Cauchy's stress tensor and p is the pore water pressure.

Equation of Motion

Equation of motion of fluid saturated soil is given by

$$\rho a_i^S + \rho^F a_i^R = S_{ji,j} + \rho b_i \tag{2}$$

where ρ is the mass density of saturated soil, ρ^F is the mass density of pore fluid, a_i^S is the acceleration vector component of fluid saturated soil, a_i^R is a relative acceleration, S_{ij} is the nominal stress and b_i is the body force.

Continuity Equation

Continuity equation is derived by the mass conservation law and the equation of pore fluid as:

$$\rho^F \dot{D}_{ii}^S - \frac{\rho^F}{n}\dot{D}_{ii}^R = p_{,ii} + \frac{\gamma_w}{k} D_{ii}^S \tag{3}$$

where dot $\bullet$ denotes time differentiation, n is the porosity, D_{ij}^S and D_{ij}^F are the stretching tensors for solid and fluid phases, respectively.

Constitutive Model

A cyclic elasto-plastic model for sand based on non-linear kinematic hardening rule (Oka et al. 1999 [14]) is used in the analysis. The features of the model are given in the following section.

Koizumi (2000) [15] and Oka et al. (2001) [16] developed a liquefaction analysis code based on finite deformation with *u-p* formulation, *LIQCA-FD*. They applied the proposed model to the behaviors of quay wall and the foundation beneath embankment it was found that the liquefaction analysis method based on the finite deformation theory is well applicable to predict the liquefaction induced deformation. In the present study we have newly developed a new FEM code based on *u-w-p* formulation which is referred to as *LIQCA-FW*.

In order to discretize equations of motion for water saturated soil and the continuity equation for pore fluid in time, Newmark's β method and finite time difference method are used. In the formulation, acceleration vector $\{a_N\}$, relative acceleration vector $\{a_N^R\}$ and pore water pressure $\{p_N\}$ are taken as independent variables.

After manipulation, discretized governing equations are given in matrix form as:

$$[A]\{x\} = \{B\} \tag{4}$$

$$[A]=\left[\begin{array}{c|c|c} [M]_{t+\Delta t}+\gamma(\Delta t)^2\left([K]_{t+\Delta t}+[K_L]_{t+\Delta t}\right) \\ +\gamma(\Delta t)[R]_{t+\Delta t} & \dfrac{\gamma_w}{\rho g}[M]_{t+\Delta t} & [K_v]_{t+\Delta t} \\ \hline \dfrac{\gamma_w}{\rho g}[M]_{t+\Delta t} & \dfrac{\gamma_w}{\rho}\left(\dfrac{1}{ng}+\dfrac{\gamma(\Delta t)}{k}\right)[M]_{t+\Delta t} & -[K_v]_{t+\Delta t} \\ \hline [K_v]^T_{t+\Delta t} & \dfrac{1}{n-\dfrac{\gamma(\Delta t)ng}{k}}[K_v]^T_{t+\Delta t} & \dfrac{1}{\gamma_w(\dfrac{1}{g}-\dfrac{\gamma(\Delta t)}{k})}[K_h]_{t+\Delta t} \end{array}\right]$$

$$\{x\}=\left\{\begin{array}{c} \{a_N^S\}_{t+\Delta t} \\ \hline \{a_N^R\}_{t+\Delta t} \\ \hline \{p_N\}_{t+\Delta t} \end{array}\right\}$$

$$\{B\}=\left\{\begin{array}{c} \{F\}_{t+\Delta t}-\{S'\}_t-\{(\Delta t)([K]_{t+\Delta t}+[K_L]_{t+\Delta t})+[R]_{t+\Delta t}\}\{(\Delta t)(1-\gamma)\{a_N^S\}_t+\{v_N^S\}_t\} \\ +[K_v]_{t+\Delta t}\{p_N\}_t-(\Delta t)\{T_W\}_t-(\Delta t)\{T_L\}_t \\ \hline \dfrac{\gamma_w}{g}\{B_f\}_{t+\Delta t}-\dfrac{\gamma_w}{k\rho}[M]_{t+\Delta t}(\{v_N^R\}_t+(1-\gamma)(\Delta t)\{a_N^R\}_t) \\ \hline \dfrac{1}{(k/g)-(\Delta t)\gamma}[K_v]^T_{t+\Delta t}\{(1-\gamma)(\Delta t)\{a_N^S\}_t+\{v_N^S\}_t\} \end{array}\right\}$$

where $[M]$ is the mass matrix, $[K]$ is the stiffness matrix, $\{a_N\}$ is the acceleration vector at node, $\{v_N\}_{t+\Delta t}$ is the velocity vector, $\{p_N\}$ is the pore water pressure vector, Δt is the increment of time, ρ_f is the mass density of pore fluid, superscripts *S* and *R* represent solid and relative components, respectively. γ_w is the unit weight of pore water and k is the permeability coefficient.

3 Constitutive Model for Sand

In the analysis, we have extended an elasto-plastic model for sand which was derived based on the non-linear kinematic hardening theory Adachi and Oka (1982) [17], Oka et al. (1999) [14] to the model applicable for the finite deformation theory. In the finite deformation analysis, objective stress rate is necessary. As for the stress rate tensor, we used a Jaumann's stress rate tensor $\hat{T}'_{ij}$ of effective Cauchy stress tensor.

$$\hat{T}'_{ij}=C^{EP}_{ijkl}D_{kl} \tag{5}$$

where C^{EP}_{ijkl} is the elasto-plastic compliance tensor and D_{kl} is the stretching tensor.

Total stretching tensor can be decomposed into elastic (D^E_{ij}) and plastic stretching tensor (D^P_{ij}) as

$$D_{ij}=D^E_{ij}+D^P_{ij} \tag{6}$$

The model used in the analysis has three main features. The first feature of the model is that the generalized flow rule is developed to accurately describe the dilatancy characteristics of sand, and the second one is that cumulative plastic strain dependence of the plastic shear modulus is introduced in order to well simulate a cyclic mobility. The third one is the introduction of a fading memory of the initial anisotropy. The constitutive model employs an overconsolidation boundary surface which distinguishes the overconsolidated (O.C.) region from the normally consolidated (N.C.) region. The non-linear kinematic hardening variable is used in the yield function as well as in the plastic potential function.

Overconsolidation Boundary Surface

Overconsolidation boundary surface is introduced to define the N.C. and O.C. regions. The overconsolidation boundary surface, $f_b = 0$, is defined as follows:

$$f_b = \overline{\eta}^*_{(0)} + M^*_m \ln \frac{T'_m}{T'_{mb}} = 0 \tag{7}$$

$$\overline{\eta}^*_{(0)} = \left\{ (\eta^*_{ij} - \eta^*_{ij(0)})(\eta^*_{ij} - \eta^*_{ij(0)}) \right\}^{1/2} \tag{8}$$

$$\eta^*_{ij} = T^D_{ij} / T'_m \tag{9}$$

where T'_m is the mean effective stress, T^D_{ij} is the deviatoric stress tensor, M^*_m is the value of the stress ratio expressed by $\{\eta^*_{ij}\eta^*_{ij}\}^{1/2}$ when the maximum volumetric strain during shearing takes place and which could be called the phase transformation stress ratio, and $\eta^*_{ij(0)}$ denotes the value of η^*_{ij} at the end of consolidation. The condition $f_b < 0$ means that the stress state stays in an overconsolidated region (O.C. region), while $f_b \geq 0$ means that the stress state stays in a normally consolidated region (N.C. region). Herein, T'_{mb} in Eq.(7) is given as follows:

$$T'_{mb} = T'_{mbi} \exp\left(\frac{1+e}{\lambda - \kappa} v^P \right) \tag{10}$$

where T'_{mbi} is the initial value of T'_{mb}, κ is the swelling index and λ is the compression index, e is the void ratio, v^P is the plastic volumetric strain.

The Generalized Flow Rule

A stress-dilatancy characteristic derived by conventional flow rule sometimes gives a rather steeper slope to the liquefaction strength curve than that obtained from laboratory tests. In order to counteract this shortcoming in the original model, the flow rule is generalized using the fourth rank isotropic tensor, H_{ijkl} as

$$D^P_{ij} = H_{ijkl} \frac{\partial g}{\partial T'_{kl}} \tag{11}$$

$$H_{ijkl} = a\delta_{ij}\delta_{kl} + b(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{12}$$

Plastic Strain Dependence of the Plastic Shear Modulus

One of the problems with the constitutive model for sand is how well the behavior could be reproduced under cyclic mobility. To reproduce the continuous increase in shear strain under cyclic mobility for loose sand, the strain dependence of the plastic shear modulus should be taken into account.

4 Numerical Analysis

In the present study, we have numerically analyzed a dynamic behavior of homogeneous sandy ground model. Fig.1 shows a model ground for numerical analysis. The finite deformation based effective stress analysis mentioned above was performed using two methods based on both *u-p* formulation and *u-w-p* formulation. As for the finite elements, 20 noded isoparametric element for displacements of soil skeleton and relative displacements, and 8 noded isoparametric element for pore water pressure are used. 2% Rayleigh damping and $\beta = 0.3025$, $\gamma = 0.6$ are used in the analysis. The bottom and lateral surfaces are assumed to be impermeable boundaries. Hence, relative displacements (w_i) between two phases, w_x and w_y are zero: $w_x = 0$ ($x = 0$, 3 m), $w_y = 0$ ($y = 0$, 3 m). At the bottom, all of the displacements are fixed: $u_i = 0$, $w_i = 0$, $i = x, y, z$.

The 20 second of horizontal *sine* wave with maximum acceleration of 250 gal was used as input motion in the numerical analysis.

In Table 1, material parameters used in the calculation are listed. The parameters are determined for the medium dense Toyoura sands (Oka et al. 1999 [14]). Table 2 shows frequency and permeability coefficient used for the numerical analysis. In Fig.1, the nodal points are denoted where output of calculated results is taken. Fig.2 shows the maximum values of acceleration for solid phase and relative acceleration in depth during the calculation of 20 seconds. The relative acceleration is larger for high frequency and high permeability coefficient. Figs.3 and 4 are acceleration of solid phase - time profile at the nodal point A and pore water pressure - time profile at the nodal point A, respectively. In case f-2, in which the frequency of input motion and the coefficient of permeability are high, the difference between *u-p* formulation (*LIQCA-FD*) and *u-w-p* formulation (*LIQCA-FW*) is large. This means that *u-w-p* formulation is necessary to well simulate the behavior of gravel layer with high permeability.

Table 1: Material parameters

γ_w	9.8 (kN/m^3)	M_m^*	0.707
γ_{sat}	18.98 (kN/m^3)	M_f^*	0.990
K_0	0.5	B_0^*	3500
e_0	0.772	B_1^*	350
λ	0.02	C_f^*	2000
κ	0.0052	n	1.5
ν	0.25	γ_{DAr}^P	0.004
g	9.8 (m/s^2)	γ_{DAr}^E	0.001

Table 2: Frequency and coefficient of permeability used in the analysis.

Cases	Frequency of input *sine* wave	Coefficient of permeability
case s-4	0.5 (Hz)	1.0×10^{-4} (m/s)
case s-3	0.5 (Hz)	1.0×10^{-4} (m/s)
case s-2	0.5 (Hz)	1.0×10^{-4} (m/s)
case m-4	5.0 (Hz)	1.0×10^{-3} (m/s)
case m-3	5.0 (Hz)	1.0×10^{-3} (m/s)
case m-2	5.0 (Hz)	1.0×10^{-3} (m/s)
case f-4	10.0 (Hz)	1.0×10^{-2} (m/s)
case f-3	10.0 (Hz)	1.0×10^{-2} (m/s)
case f-2	10.0 (Hz)	1.0×10^{-2} (m/s)

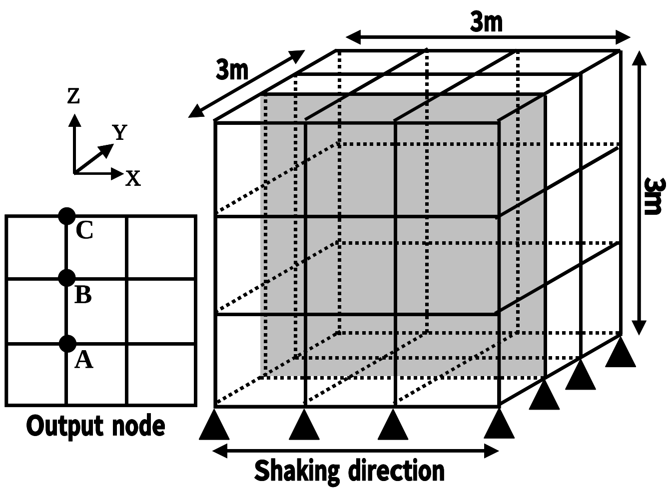

Figure 1: Model ground used in the analysis

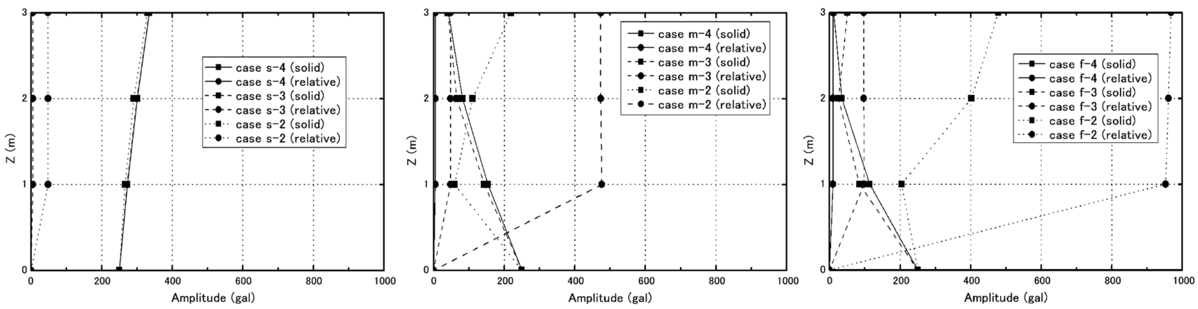

Figure 2: Maximum amplitudes of relative acceleration response

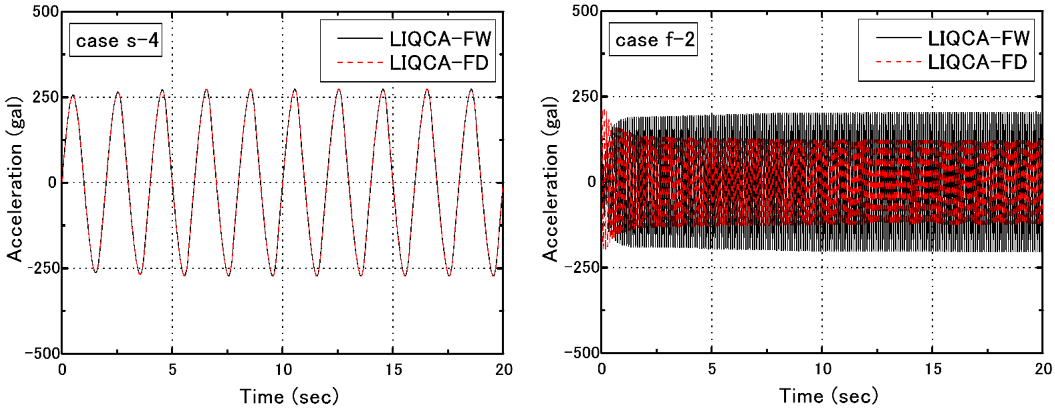

Figure 3: Acceleration of solid phase - time profile at the nodal point A

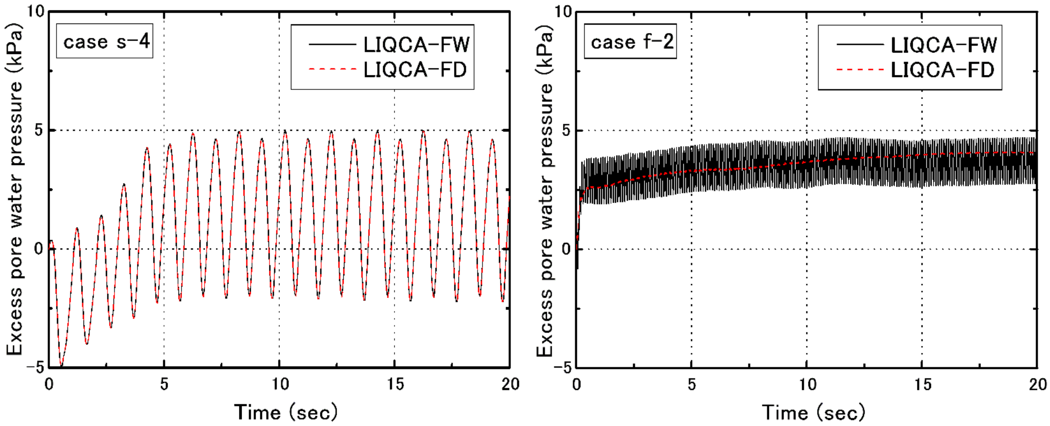

Figure 4: Pore water pressure - time profile at the nodal point A

5 Conclusions

In the present study, a liquefaction analysis method based on the finite deformation theory using *u*-*w*-*p* formulation is studied. The proposed theory was derived based on the updated Lagrangian method and the Jaumann stress rate tensor. The proposed method is compared with the results by *u*-*p* formulation. We have found that the difference between results by two methods is large in the case of high permeability coefficient and high frequency of input motion.

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