Deformation and stability analysis of rectangular tunnel in soft rock ground using a strain softening type elasto-plastic model

T.Adachi, F.Oka, T.Kodaka & J.Takato
Kyoto University, Kyoto, Japan

ABSTRACT: The purpose of this study is to investigate the deformation and stability of rectangular tunnel excavated in the soft rock ground. In order to simulate the excavation of a rectangular tunnel below the underground water level, a series of the soil-water coupled finite element analyses using a strain softening type elasto-plastic constitutive model is carried out. The generally expected negative excess pore water pressure hardly occurs around the rectangular tunnel in the given soft rock ground due to the unloading by the excavation. The unstable behavior caused by the dissipation of the negative pore water pressure cannot be observed. Furthermore, it can be confirmed that the additional settlement of ground surface due to the drainage does not occur.

1 INTRODUCTION

Throughout most of Japan, soft rock ground not only occurs in mountain areas, but also in the relatively shallow subsurface of urban areas. Considering the profuse utilization of underground space in urban areas for expressways, parking lots, and storage areas, the number of excavations using rectangular tunnels is expected to rise. In the present study, both the deformation and stability characteristics of a rectangular tunnel excavated in submerged soft rock ground are discussed. For this purpose, a series of soil-water coupled finite element analyses is carried out. The constitutive model used in the computations is the strain softening, elasto-plastic constitutive model proposed by Oka & Adachi (1985) and Adachi & Oka (1992, 1995). First of all, the derivation of the constitutive model is outlined. Since the stability of submerged underground tunnels is being studied, we conduct a soil-water coupled analysis with a strain softening type of constitutive model. In this present study, the progressive enlargement of the tunnel section into various sizes is made in order to investigate the stability of the rectangular tunnel.

2 STRAIN SOFTENING TYPE ELASTO-PLASTIC CONSTITUTIVE MODEL

Oka and Adachi 1985, Adachi and Oka 1992, 1995, proposed an elasto-plastic constitutive theory for soft rocks by introducing the stress-history tensor with respect to the strain measure. They assumed that the material strength of soft rocks was composed of frictional strength, and a component due to cementation and/or the cohesions of materials as shown in Figure 1. The main factor responsible for strain softening after peak strength is the decrease of cementation and/or cohesion strength component. This can be seen to be due to the degradation of internal structure.

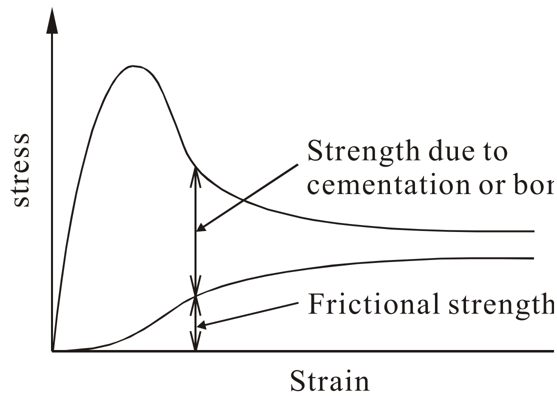


Figure 1. Schematic diagram of stress-strain relation with strain softening.

2.1 *Stress history tensor*

Stress history tensor is expressed by

$$\sigma_{ij}{}^* = \int_0^z K(z-z')\sigma_{ij}(z')dz' \tag{1}$$

$$dz = \sqrt{de_{ij}de_{ij}}$$

where z is a strain measure which is the second invariant of deviator strain, and K is the continuous bounded kernel function with the assumption that $\partial K / \partial z < 0$. In this study, the simple exponential function is adopted for soft rock, i.e.

$$K(z) = \exp(-z/\tau)/\tau \quad (2)$$

where τ is a strain softening material parameter. Substituting Eq.(2) into Eq.(1), the stress history tensor is described as

$$\sigma_{ij}* = \frac{1}{\tau}\int_0^z \exp(-(z-z')/\tau)\sigma_{ij}(z')dz' \quad (3)$$

2.2 *Non-associated flow rule and yield function*
The following non-associated flow rule is adopted:

$$d\varepsilon_{ij}^p = H\frac{\partial f_p}{\partial \sigma_{ij}}df_y \quad (4)$$

where f_p is the plastic potential function, f_y is the yield function and H is the hardening-softening function. The yield function f_y does not directly depend on the current stress state, but depends on stress history ratio and strain hardening-softening parameter κ:

$$f_y = \eta^* - \kappa = 0\,, \quad \eta^* = \sqrt{(s_{ij}^* s_{ij}^* / \sigma_m^{*2})} \quad (5)$$

where s_{ij}^* is the deviatoric part of the stress history tensor σ_{ij}^*, and σ_{ij}^* is the isotropic part of the stress history tensor.

2.3 *Strain hardening-softening parameter*
The evolution equation of the strain hardening-softening parameter κ is expressed as follows:

$$d\kappa = \frac{G'(M_f^* - \kappa)^2}{M_f^*}d\gamma^p \quad (6)$$

where M_f^* is the value of η^* at the residual state. By integration under proportional loading conditions with zero as an initial value of κ, the following hyperbolic function is given:

$$\kappa = \frac{M_f^* G'\gamma^p}{M_f^* + G'\gamma^p} \quad (7)$$

in which the plastic shear strain is defined in the following equation, i.e.

$$\gamma^p = \int d\gamma^p = \int (de_{ij}^p de_{ij}^p)^{1/2} \quad (8)$$

Then, the strain hardening- softening parameter κ is given by

$$\kappa = \int d\kappa \quad (9)$$

The strain hardening-softening parameter G' is the initial tangent of the hyperbolic function given in Eq.(7).

2.4 *Plastic potential function and overconsolidated boundary surface*
The overconsolidated boundary surface f_b is defined as the limit shape of plastic potential function f_p, and denotes as follows:

$$f_b = \overline{\eta} + \overline{M}_m \ln[(\sigma_m + b)/(\sigma_{mb} + b)] = 0 \quad (10)$$

where $\overline{\eta}$ is the stress ratio defined by

$$\overline{\eta} = \sqrt{s_{ij}s_{ij}/(\sigma_m + b)^2} \quad (11)$$

The overconsolidated boundary surface parameter $\overline{M}_m$ is the value of $\overline{\eta}$ when maximum volumetric compression occurs. Material parameters b and σ_{mb} describe the material internal structure. Adachi et al. (1998) assumed that σ_{mb} changes with the value of plastic volumetric strain as follows:

$$\sigma_{mb} = \sigma_{mb0}\exp\left(\frac{v^p}{\lambda_p - \kappa_p}\right) \quad (12)$$

where σ_{mb0} is the initial value of σ_{mb}, and equivalent to the consolidation yield stress. The compression and swelling indexes are denoted by λ_v and κ_v, respectively.

The plastic potential function for soft rock is given by

$$f_p = \overline{\eta} + \overline{M}\ln[(\sigma_m + b)/(\sigma_{mb} + b)] = 0 \quad (13)$$

where $\overline{M}$ is defined as follows:
In the overconsolidated region $(f_b < 0)$,

$$\overline{M} = -\overline{\eta}/\ln[(\sigma_m + b)/(\sigma_{mb} + b)] \quad (14)$$

In the normally consolidated region $(f_b \geq 0)$,

$$\overline{M} = \overline{M}_m \quad (15)$$

2.5 *Strain softening type elasto-plastic constitutive equation*
The plastic strain increment tensor, $d\varepsilon_{ij}^p$, is derived using Prager's compatibility condition in the following Eq. (16) together with Eqs.(4), (7) and (13), i.e.

$$df_f = d(\eta^* - \kappa) = 0 \quad (16)$$

$$d\varepsilon_{ij}^p = \Lambda\left[\frac{\overline{\eta}_{ij}}{\overline{\eta}} + (\overline{M} - \overline{\eta})\frac{\delta_{ij}}{3}\right]\left[\frac{\eta_{kl}*}{\eta^*} - \eta^*\frac{\delta_{kl}}{3}\right]\frac{d\sigma_{kl}*}{\sigma_m^*} \quad (18)$$

$$\Lambda = \frac{M_f{}^{*2}}{G'(M_f{}^* - \kappa)^2} \quad (19)$$

Consequently, the incremental elasto-plastic constitutive equation can be obtained by adding the plastic strain increment tensor in Eq.(18) to the elastic strain increment tensor.

3 ANALYSIS CONDITIONS

The details of soil-water coupled finite element analysis with a strain-softening elasto-plastic model can be found in Adachi et. al. (2000). The finite element analysis is carried out using four-noded isoparametric elements under the plane strain conditions. At the center of each element, excess pore water pressure (or total head) is defined, while a spatial discretization of continuum equations for the water phase is performed using the finite difference method proposed by Akai and Tamura, 1978.

Table 1. Material parameters used in the analysis

Young's modulus $E(MN/m^2)$	200.0
Poisson's ratio ν	0.33
Submerged unit weight $\gamma'(kN/m^3)$	12.8
Permeability $k(cm/\mathrm{sec.})$	10^{-5}
Strain hardening-softening parameter G'	612.2
Strain hardening-softening parameter M_f^*	1.0
Plastic potential parameter $b(kPa)$	160
Overconsolidation boundary parameter $\overline{M}_m$	0.95
Overconsolidation boundary parameter $\sigma_{mb}(MPa)$	1.47
Stress history parameter τ	0.04

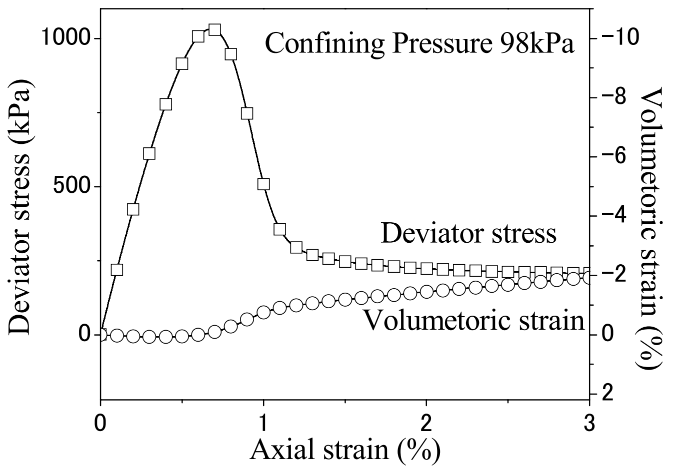


Figure 2. Simulation result of triaxial compression test under drained conditions.

Table 1 shows the material parameters used in the analysis. These parameters, in fact, correspond to a weathered sandstone that is in the Kansai area in Japan. Figure 2 shows a simulated stress-strain curve for a drained triaxial compression test by the constitutive model with the parameters given in Table 1. In the analysis, the confining pressure dependency of material parameters has not been considered, and constant values were used for each one of them for simplicity. Figure 3 illustrates the finite element mesh used together with the associated boundary conditions. The total numbers of elements and nodes are 1750 and 1836, respectively.

Firstly, an initial stress distribution of ground prior to excavation is calculated. Then equivalent nodal forces for elements to be reduced due to excavation of tunnel are calculated from the initial stress distribution.

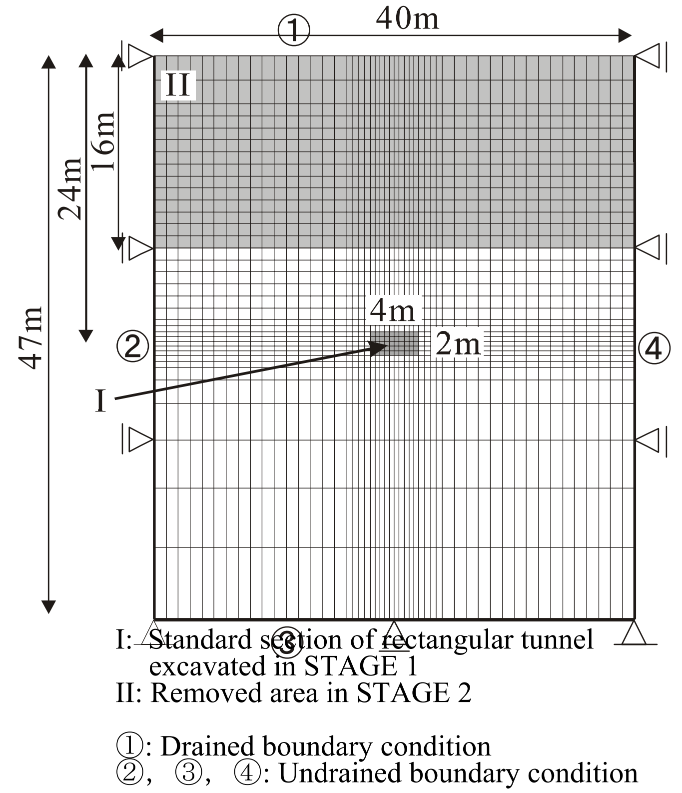


Figure 3. Finite element array and boundary conditions used in the analysis.

Calculations are carried out by the following four stages of analysis

STAGE 1: A standard rectangular section tunnel whose center is located at a depth of 24 m is excavated with a width of 4 m and height of 2 m.

STAGE 2: A 16 m thick upper layer of soft rock ground is removed to decrease the overburden pressure of the tunnel. This stage of analysis is introduced to discuss the stability of the standard sectional rectangular tunnel in the case that the overburden pressure is rapidly decreased.

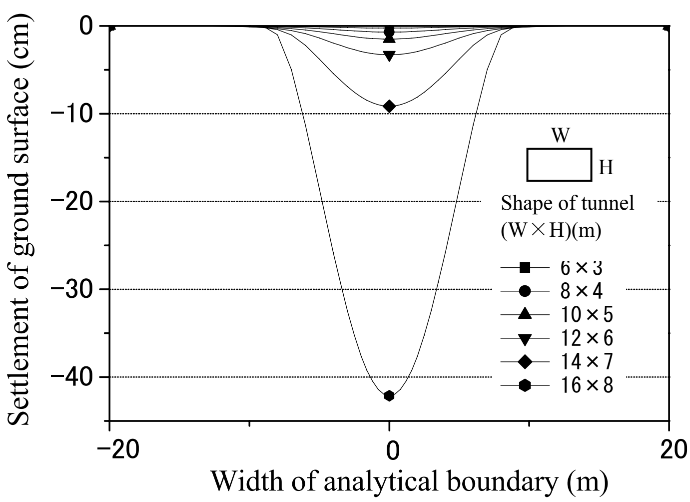
STAGE 3: The tunnel is enlarged to investigate the stability of the standard rectangular section tunnel excavated in STAGE 1. In order to evaluate the stability of the opening, the sectional area of the tunnel is gradually increased. The release rate of stress around the opening is 1% of excavation stress per step with a total number of 100 steps used during the enlarging of the tunnel section. The calculation time period for each step is 1000 seconds. During the enlargement process, it is assumed that the water level is at the ground surface, and the inside of the tunnel is filled with underground water. It is more reasonable to study the stability of the tunnel by considering various sizes of tunnel instead of increasing body forces until collapse is reached for a fixed opening size. Furthermore, a total stress based analysis is also carried out for comparison with an effective stress based analysis.

STAGE4: The underground water that fills the tunnel is forced to drain so that total stress on its boundaries is zero. The stability of the rectangular tunnel is discussed as the internal pressure of the tunnel is decreased from hydrostatic water pressure to atmospheric pressure levels (zero total stress).

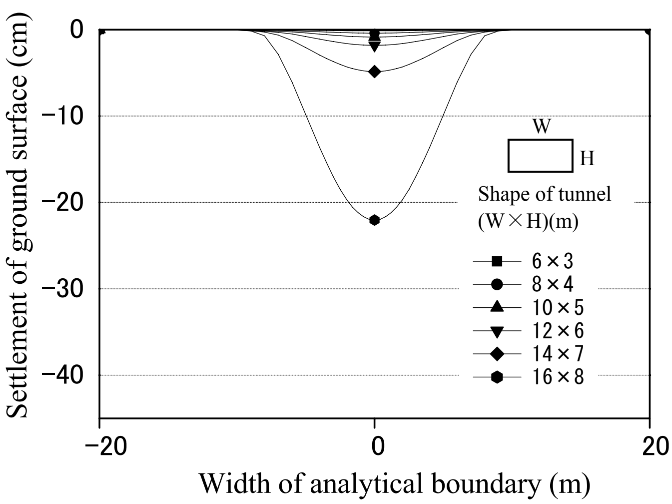
4 DISCUSSION OF RESULTS

Since displacements observed in both STAGE 1 and 2 are small, only computational results obtained in STAGE 3 and 4 are discussed in this section.

Figure 4 shows the settlement of ground surface after the enlargement of the tunnel opening into various sections in STAGE 3. The ground surface is redefined after removing the 16m thick layer in STAGE 2. Figs. 4a and 4b. show the ground surface settlements calculated for various tunnel sections using both total and effective stress analyses. The settlements obtained from the effective stress analysis is smaller than the ones from a total stress analysis because in the former, submerged conditions were assumed. Figure 5 shows the excess pore water pressure, stress history ratio, shear stress and plastic shear strain which are calculated just after the excavation of a 14 m x 17 m rectangular tunnel section in STAGE 3. The results after one week and 10 days are also shown. When the tunnel section is enlarged during excavation, both vertical and lateral stresses are unloaded. In the case that a shear stress due to unloading applies to heavily overconsolidated soft rock, negative excess pore water pressures is expected to occur. However, negative excess pore water pressures hardly develop around both the crown and the sidewall of the tunnel. Also, the changes in shear stress, plastic shear strain, and stress history ratio during the period after excavation are indeed very small. Consequently, the rectangular tunnel assumed in this analysis does not become unstable due to the dissipation of negative pore water pressures. Figure 6 shows the settlements of the ground surface

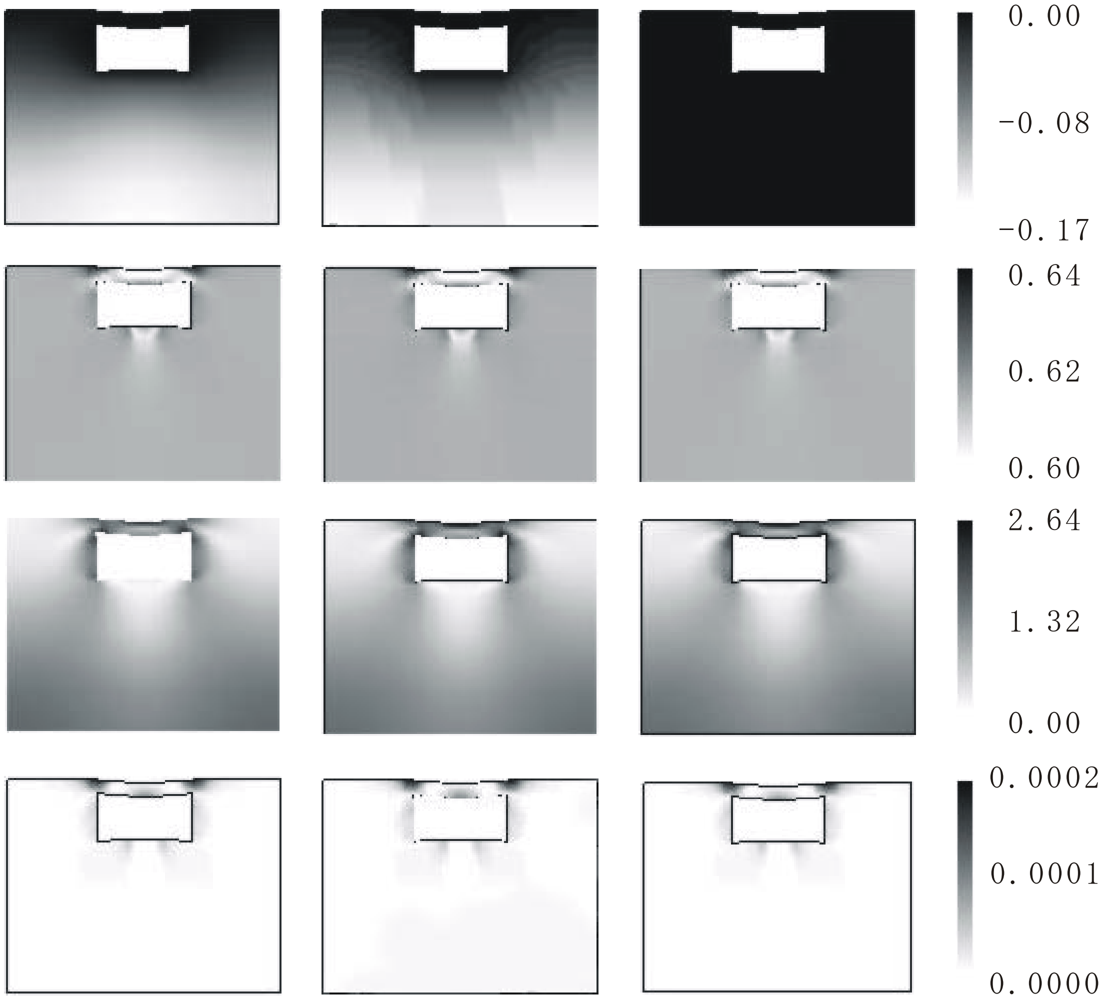


(a) Total stress based analysis



(b) Effective stress based analysis

Figure 4. Settlements of ground surface during the excavation of various rectangular section tunnels.



with different sections of the tunnel excavated in STAGE 4. There are hardly any changes in settlements when comparing Fig.4b with Fig.6, which implies that the drainage of water outside the tunnel does not affect the settlement of the ground surface or the stability of the opening. Figure 7 illustrates the total water head distribution after the drainage. The drop in total water head is observed only around the tunnel. It should be noted that this result is limited to the special condition under which the underground water is considered abundant and there is no water level drawdown. In the case where there is water level drawdown due to drainage, results may be different.

Shape of tunnel
(W×H)(m)

Figure 6. Settlements at ground surface during drainage (STAGE 4)

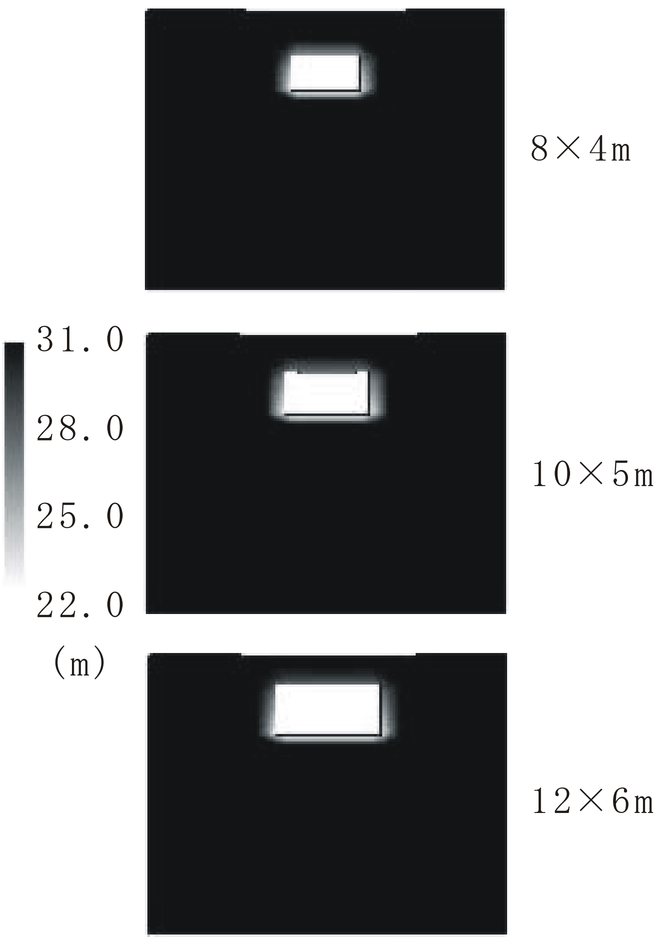


Figure 7. Distributions of total water head with different rectangular section tunnels during drainage (STAGE 4)

5 CONCLUSIONS

In order to study the stability of rectangular tunnels excavated in soft rock ground, a series of soil-water coupled finite element analyses using a strain softening elasto-plastic constitutive model has been performed. It was found that generally expected negative pore water pressures due to excavation hardly develop around the rectangular tunnel in the given soft rock ground. Hence no tunnel instability was observed. Furthermore, it was confirmed that no additional surface settlement due to drainage occurred. The relative length of lateral boundaries against an opening of tunnel may influence the obtained results. However, it can be expected that the results obtained with the shorter lateral boundaries are severer against a collapse of tunnel.

REFERENCES

Adachi, T. and F. Oka 1992. An elasto-plastic constitutive model for soft rock with strain softening. *Proc. of JSCE, Journal of Geotechnical Engineering*. 445/III-18: 9-16. (in Japanese)

Adachi, T. and F. Oka 1995. An elasto-plastic constitutive model for soft rock with strain softening. *Int. J. Numerical and Analytical Methods in Geomechanics*: 19, 233-247.

Adachi, T., F. Oka, A. Koike and M. Koike 1998. Modified Adachi-Oka's elasto-plastic constitutive model for soft rock. *Proc. of JSCE, Journal of Geotechnical Engineering*. 589/III-42: 31-40. (in Japanese)

Adachi, T., F. Oka, H. Osaki and F. Zhang 2000, Soil-water coupling analysis of progressive failure in cuts with a strain softening model, Proc. Constitutive Modelling of Granular Materials: 471-490.

Akai, K. and T. Tamura 1978. Elasto-plastic numerical analysis of soil-water coupling problem, *Journal of JSCE*: 269: 95-104. (in Japanese)

Oka, F. and T. Adachi 1985. A constitutive equation of geologic materials with memory. *Proc. of Int. Conf. on Numerical Methods in Geomechanics*, 1, 293-300.